BASIS OF RELIABILITY DESIGN

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Abstract
The knowledge and experience learnt from product designing have resulted in development of their reliability theory. The classical concept of safe – life is based on product over dimensioned design that considers safety factor or safety margin for measure. However, practical engineering has found this concept in a manner inconvenient as design fault-resistance to determine the ultimate condition and operating stress are random values. A way out is in the concept of stochastic approach to reliability design resulting from the defect-production probability distribution law. That concept allows product designing with predetermined reliability, such as in the example contained in the paper.

Keywords: design classical approach, safety factor, design stochastic approach, safe designing.

1 Introduction

There are undoubtedly objective principles about designing reflected in both the creative procedure and possible algorithmisation or automation of each of the processes. Algorithmisation then stands for a general procedure of multiple steps leading to the result of the questioned task. Algorithmisation of designing is a systematic method possibly completed for practical effect with intuitive methods using the designer's or the creative individual's education, knowledge and experience bases for the results. The systematic and intuitive kind methods do complete rather than inhibit from each other that makes us think of a new design discipline – Design Science.

The first phase of a new equipment design project is dimensioning process of setting up the mechanical parts size supporting cross-section dimensions so that they stand the external load. It is practical a priori determination on design requirements of strength, rigidity and other features that include also reliability at reasonable production cost. The current practice in designing has determined the essential parameters and functions to assess the load strength of the material by modelling single-value deterministic variables. The errors of calculation, if any, material defects, inaccurate knowledge of loading forces, oscillating load cycles and others are considered within the safety factor "*k*". The stress-restricted designs have the acceptance criterion in the following form:

$$S \geq s\,.\,k\,, \qquad (1)$$

where $k > 1$ … for the safety factor.

However, safety induced in such a way to the design misses information of the construction element failure option in the form of failure probability F(t). This conventional attitude supposes that sufficiently high safety factor *k* may completely eliminate the failure possibility. However, most design variables are actually random values that temper with the probability theory and mathematic statistics principles. Therefore, the failure probability F(t) may vary a lot depending on element load and strength random variables distribution at equal safety factor *k*.

Should the machine constructions design reflect their reliability the design formulas should consider stochastic character of all the variables and use all the terms for the general design. This attitude to designing called reliability (probability) design has developed only recently. Its suitability for aircraft or automotive industries has increased as reliability of the product, such as aircraft or automobile has become an important factor besides economic operation in the competitive environment of the global market.

2 Conventional and reliability-based approach to construction design

Reliability designing is based on the presumption the design variables are random values. Their variability rate may be used for attaining the required construction reliability. The difference of the reliability-based and conventional (deterministic) design may be explained on the design example of strength S intended to transmit static service loads.

The conventional approach to design introduces safety factor k > 1 and according to the equation (1) the design criterion takes the following form:

$$\frac{S}{s_{\max}} = k \,, \qquad (2)$$

where S … characteristic material strength (such as yield point, ultimate strength, endurance limit and others),

s_{max} … maximum permissible stress (such as tension etc.).

The actual operating stress must comply the following condition

$$s \leq s_{max} = \frac{S}{k} \rightarrow S - s \geq k \,, \qquad (3)$$

to guarantee failure is avoided. Failure occurrence probability cannot be reflected in the formula or specify with a value.

The reliability design approach reflects reliability or design safety in the form of failure-proof probability. The design criterion is considered in the following form

$$S > s \rightarrow (S - s) > 0. \qquad (4)$$

Probability introduced to the inequation results in the following

$$P\,(S > s) = P\,(S - s) = R \,, \qquad (5)$$

Where $S = (\hat{S}, \sigma_S)$ … characteristic material strength,

$S = (\hat{s}, \sigma_s)$ … maximum accessible operating stress,

R … probability of failure-proof operation, i.e. structural reliability.

The equation (5) may be expressed in words that due to probability R material strength is higher than the acting tension caused by the operating load and prevents the failure. The difference of the conventional and reliability approaches is characterised in figures 1 and 2.

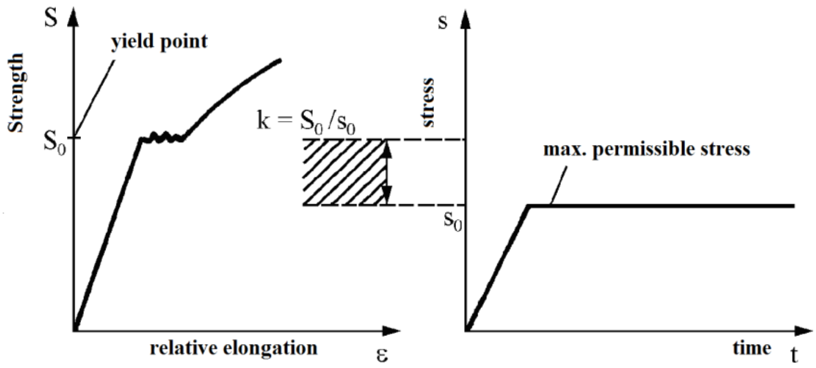

Fig. 1: *Conventional design*

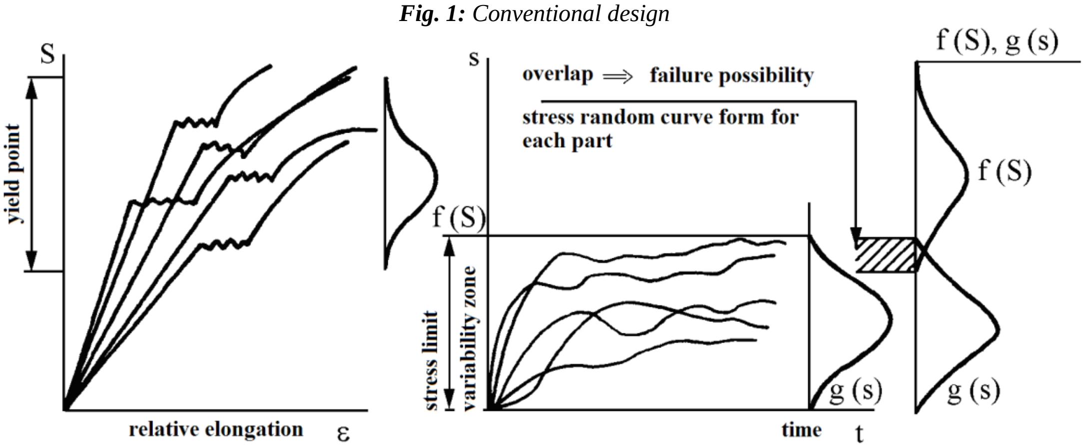

Fig. 2: *Reliability design*

3 Reliability designing and analysis

If a part strength S = (μ_s , σ_s^2) considered when the random values are distributed along normal (Gauss) rule N (μ , σ^2) where μ , σ^2 are real numbers, $\sigma^2 > 0$ thus

$$f(S) = \frac{1}{\sigma_s \sqrt{2\pi}} \exp - \frac{(S-\mu_s)^2}{2\sigma_s^2} \qquad (6)$$

while S $\in (-\infty, +\infty)$, or ($-\infty < S < \infty$).

This part is randomly loaded with stress s = (μ_s , σ_s^2) with normal distribution, then

$$f(s) = \frac{1}{\sigma_s \sqrt{2\pi}} \exp - \frac{(s-\mu_s)^2}{2\sigma_s^2} \qquad (7)$$

and ($-\infty < s < \infty$).

Equation (5) implies R = P (S – s > 0) and the difference is z = S – s with the following characteristics:

$$\mu_z = \mu_S - \mu_s \qquad (8)$$

and

$$\sigma_z = \sqrt{\sigma_S^2 + \sigma_s^2} \qquad (9)$$

then

$$R = \frac{1}{\sigma_z\sqrt{2\pi}} \int_0^{\infty} \exp - \frac{(z-\mu_z)^2}{2\sigma_z^2}\, dz . \qquad (10)$$

This equation is used in the form of standard (basic) normal distribution utilising the known transformation

$$t = \frac{z-\mu_z}{\sigma_z} \qquad (11)$$

that implies the following for the variable t limits: z = ∞ → t = ∞ and z = 0 → t = $\frac{\mu_z}{\sigma_z}$.

Failure-proof operation probability then is

$$R = \frac{1}{\sqrt{2\pi}} \int_{\frac{\mu_z}{\sigma_z}}^{\infty} \exp^{-\frac{t^2}{2}} dt = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{\mu_z}{\sigma_z}} \exp^{-\frac{\mu_z}{\sigma_z}} dt = \Phi(\frac{\mu_z}{\sigma_z}). \qquad (12)$$

The upper limit value of t is determined as follows

$$t = \frac{\mu_z}{\sigma_z} = \frac{\mu_S - \mu_s}{\sqrt{\sigma_S^2 + \sigma_s^2}} \qquad (13)$$

The equation (12) is the distribution function of the standard random variable $\Phi(t)$ with normal distribution N(0, 1). The values of the standard random variable t distribution function $\Phi(t)$ are called Laplace's functions (probability integral) being tabled.

There are two general approaches to the reliability calculation:

1.) For the already produced and used constructions, substitution of the known values of (μ_S, σ_S) and (μ_s, σ_s) in the equation (13) determines the upper limit value of t for the analysed construction and with the standard normal distribution N (0, 1) tables specify the questioned construction's reliability. The following analysis example shows the procedure:

An automobile constructional part is loaded with static stress of normal distribution N (μ_s = 135 MPa and σ_s = 11.6 MPa) and its strength is N (μ_S = 184 MPa and σ_S = 21.5 MPa). The task is determination of the part defect probability.

The equations (8), (9) and (11) imply

$$\mu_z = \mu_S - \mu_s = 184 - 135 = 49 \text{ and } \sigma_z = \sqrt{(21.5)^2 + (11.6)^2} = 24.4 \text{ then } t = \frac{49}{24.4} = 2$$

and the standard normal distribution function tables show that the automobile part reliability R(t) = 0.9773 and failure probability F(t) = 1 – R(t) = <u>0.0227</u> .

2.) The design of new parts issues from the aprior requirement of reliability R of the structure. When the required t value is known, the constructional variables are set so that the required reliability is achieved. This procedure is the basic tool for designing parts of in-advance-determined reliability, which is the principal purpose of mechanical parts reliability designing. The following is an example of the procedure.

Remark: *The examples include only static stress with forces constant in time to make calculation easy and clear. However, practical calculations should consider mostly forces variable in time – deterministic (load-time history is determined by function that can be mathematically described, such as cyclic – symmetric, discontinuous, pulsating, asymmetrical or others) and stochastic (random) that cannot be described exactly by mathematics but using rather statistic assessment.*

A simple automobile constructional part (rod) is only loaded with static axial load Q of normal distribution N ($\mu_Q = 12.7 \,.\, 10^4$ N and $\sigma_Q = 1.8 \,.\, 10^4$ N). The rod is made from steel with the yield point in tension normally distributed N ($\mu_X = 350$ MPa and $\sigma_X = 35$ MPa. The design requires failure probability $F = 10^{-5}$, i.e. R = 0.99999. The rod is made in an automatic production process where the resulting rod cross-section A is known to be a random variable of the coefficient of variation $v_A = 0.05$, which implies the mean-root-square error $\sigma_A = 0.05\ \mu_A$. The designer is now to calculate the minimum nominal cross-section of the rod μ_A that would comply the reliability requirement R = 0.99999.

The solution is to use the equation (11) for unknown nominal cross-section μ_A. First, tension of the rod should be determined with the equation $Y = \dfrac{Q}{A}$ [N/cm^2] if the rod cross-section is defined in cm^2. The randomly variable tension of the rod whose value is Y shall have the mean value

$$\mu_Y = \frac{\mu_Q}{\mu_A} = \frac{12.7.10^8}{\mu_A} \qquad [\text{ Pa }]$$

and the mean-root-square error

$$\sigma_Y = \frac{(\mu_Q^2\sigma_A^2 + \mu_A^2\sigma_Q^2)}{\mu_A^4} \rightarrow \sigma_Y^2 = \frac{1.91.10^8}{\mu_A} \qquad [\text{ Pa }]$$

The tension in the rod follows approximately normal distribution N ($12.7/\mu_A$, $1.91/\ \mu_A$) . 10^8 Pa. The standard normal distribution function tables show t = 4.265 at R = 0.99999. Substitution in the equation (13) results in

$$4.265 = \frac{3.5 - \dfrac{12.7}{\mu_A}}{\sqrt{(0.35)^2 + (\dfrac{1.91}{\mu_A})^2}} \;.$$

When this equation is solved for μ_A the required minimum cross-section $\mu_{A\ min.} = 7.63$ cm^2 and its mean-root-square error $\sigma_A = 0.05\ \mu_A = 0.3815$ cm^2 .

4 Conclusion

Reliability designing with variables actually represented by statistic models that uses construction failure-proof operation probability R rather than the conventional safety factor offers tools to solve numerous constructional key problems. First of all, it is systematic analysis of reliability for the questioned constructional element and last but not least the reliability design including the aprior reliability requirements. There is a very close relation between this and optimum construction as the determination of optimum dimensions may enhance with the determination of optimum production and material costs. A simple example showed the procedures of reliability designing for static load with normal distribution of random variables. It should be noted that the basic procedures remain the same also for construction design when load is dynamic in character and the random variables take more complex forms of distribution. Of course, the equations for calculation are more complex then. The world leading automobile industry companies have proved the advantage of reliability design in case of large scale production as the higher cost of reliability design pay shortly back in saved material, energy, labour and also in the increased reliability rate of the final product. Other advantages of this method include efficient planning of spare parts, maintenance and more.

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