

# DEPENDENCE OF APPROXIMATION WITH THE MAXIMUM DESIGN PRESSURE TO THE YIELD STRENGTH OF THE MATERIAL AND THE APPLICABILITY OF VARIOUS TYPES OF CALCULATIONS TO THE MINIMUM SAFETY WALL THICKNESS OF THE GUN BARREL, CALCULATIONS FOR CALIBER 50BMG

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## Abstract:

*For every weapon system, safety is essential for the users. The security of small caliber weapons is associated to the firing sequence. The danger spots are the closing mechanism and its fortress of fire. Another essential safety feature of the use of small caliber weapons is the correct oversizing of the gun barrel of both standard and tormentational ammunition, especially in the case of a shot. And in this work, we will deal with the correct calculations of the minimum wall thicknesses of the barrel.*

## 1 Introduction

When designing a new gun barrel on a small caliber weapon, safety is essential. But usually there is the requirement for security and the least possible weight of the barrel, while these two requirements are in conflict. Of course, there are other important requirements if we need a barrel which, in addition to the required service life, has the best possible properties for the accuracy of long-distance shooting. It requires longer barrel as well as bigger outer diameter of the barrel. If we were designing a barrel for an automatic weapon, we would have to take into account the cooling of the barrel, what usually also means a further increase in outer diameters and also weight. When we set the requirements for what type of weapon, we are going to design the barrel, we have chosen repeating hunting rifle, repeating rifle for precision shooting "sniper rifle", semi auto or full auto. We have to rely on internal ballistics calculations. When we have internal ballistics diameters along the entire length of the barrel and the

parameters of the material that we are going to use for the production of the gun barrel, we can use suitable types of calculations to determine the minimum averages in each part of the barrel.

## 2 Experimental details

To calculate internal ballistics, we used the QuickLOAD program, which calculates internal ballistics based on input data, which are:

- cartridge – .50 BMG (12.7x99 mm)/CIP standard,
- gun barrel length 800 mm (including cartridge chamber),
- gun powder type LOVEX D100, producer Explozia Pardubice (CZ),
- amount of gun powder 14.3 g = 220.7 grains/reloading tables Explozia Pardubice (CZ),
- total maximum cartridge length .50 BMG – 138.31 mm/CIP standard,

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- usable cartridge content .50 BMG – 15,809 cm<sup>3</sup>/QuickLOAD,
- bullet Hornady A-MAX 50cal. 750 grains/item 5165 Hornady.

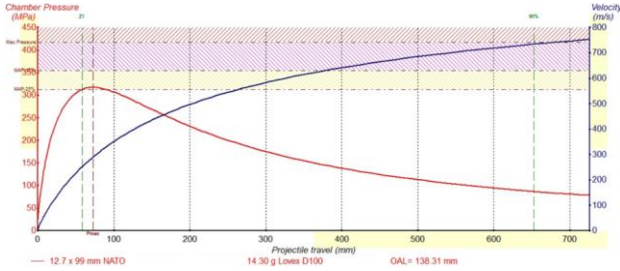


Fig. 1 Graphical representation of the course of pressure profile in the QuickLOAD program (red), the velocity of the projectile bottom (blue) and in the barrel (without the cartridge chamber) at 20 °C

In Fig. 1, we can see a graph of the course of the barrel pressure in dependence on the barrel length, where is a sharp rise in pressure to maximum pressure and a gradual decline of the barrel pressure after reaching maximum. This graph also shows the acceleration of the projectile in the barrel depending on the length of the barrel. Speed and pressure are marked from the bottom of the projectile. The bottom of the projectile is 73.808 mm from the bottom of the barrel.

We made the calculation at the following places in the barrel:

- bottom of the cartridge chamber, inner diameter at least 20.52 mm, safety coef.  $k = 1.1$ , design pressure 529.1 MPa,
- cartridge case top, inner diameter at least 14.33 mm, safety coef.  $k = 1.1$ , design pressure 529.1 MPa,
- place of maximum pressure, inner diameter 13 mm, safety coef.  $k = 1.2$ , design pressure 577 MPa,
- place at the end of the barrel, inner diameter 13 mm, safety coef.  $k = 1.2$  (safety coefficient at the end of the barrel is stated at least 2.5 in the literature [6], but since the condition of the minimum wall of the barrel is 2.5 mm, this condition of the safety coefficient is exceeded many times), design pressure 123.24 MPa, or at least the thinnest wall 2.5 mm, because the calculated size is usually smaller.

### 3 Results and discussion

Demonstration of each type of calculation of a simple unstrengthen barrel at material strength  $Re = 1100$  MPa

#### 3.1 Method 1. Calculation performed in the flexible area with sample calculation, according to the authors M. Fišer and S. Procházka, at $Re = 1100$ MPa

Dynamic value of yield strength:

- at  $Re = 1100$  Mpa
- the calculated value is  $\sigma_{KD} = 1133$  Mpa
- $p = p_k$  – inner pressure

$$\alpha = \left( \frac{\sigma_{KD}}{p_k} \right)^2$$

- at  $p_k = 529.10$  MPa the calculated value of the coefficient  $\alpha = 4.59$
- at  $p_k = 577$  MPa the calculated value of the coefficient  $\alpha = 3.86$
- at  $p_k = 123.24$  MPa the calculated value of the coefficient  $\alpha = 84.52$

$$x = \sqrt{\frac{\alpha + \sqrt{4\alpha - 3}}{\alpha - 3}}$$

- at  $p_k = 529.10$  MPa,  $x = 2.315$
- at  $p_k = 577$  MPa,  $x = 2.936$
- at  $p_k = 123.24$  MPa,  $x = 1.123$
- $d_{2MN}$  – minimum outer diameter of the barrel
- $r_1$  – inner radius of the barrel
- $d_1$  – inner diameter of the barrel

$$d_{2MN} = d_1 \sqrt{\frac{\alpha + \sqrt{4\alpha - 3}}{\alpha - 3}}$$

$$d_{2MN} = d_1 \cdot x$$

P.č.	$r_1$ (mm)	$d_1$ (mm)	$d_{2MN}$ (mm)	$\alpha$	x	p (MPa)
1	10,40	20,80	47,52	4,59	2,32	529,1
2	7,165	14,33	33,18	4,59	2,32	529,1
3	6,50	13,00	38,18	3,86	2,94	577,0
4	6,50	13,00	14,60	84,52	1,12	123,24

Fig. 2 Calculated minimum outer diameters of the barrel by Method 1 at  $Re = 1100$  MPa

This method is suitable for calculation when there is a big difference between the design pressure and the pressure of the yield strength of the material. The yield strength is much higher than the design pressure, that is when the value of the coefficient  $\alpha$  is much more than 3. If the coefficient  $\alpha$  is close to 3, the calculated values are unrealistically large. This can be eliminated by replacing the material with a better material with higher yield strength:

$$\sigma_{red}^I = \frac{2}{3} \cdot \frac{2 \cdot x^2 + 1}{a^2 - 1} \cdot p_1$$

- where  $a = r_2/r_1$  and  $x = r_2/r$ ,
- $r_1$  – inner radius of the barrel,
- $r_2$  – outer radius of the barrel,
- $r$  – radius of the barrel at the point of calculation,
- $\sigma_{red}$  = stress according to maximum elongation,
- $p = p_1$  – inner pressure.

When calculating the minimum wall thickness of the barrel, the yield strength will not be exceeded further than 0.5 mm from the inside of the barrel wall.

P.č.	$r_1$ (mm)	$r_2$ (mm)	$d_{min}$ (mm)	$r$ (mm)	$r-r_1$ (mm)	$a$	$x$	$p$ (MPa)	$s_{red}$ (MPa)
1	10.40	18.60	37.20	10.90	0.50	1.79	1.71	529.1	1094.77
2	7.165	12.50	25.00	7.665	0.50	1.74	1.63	529.1	1090.68
3	6.50	12.00	24.00	7.00	0.50	1.85	1.71	577.0	1098.53
4	6.50	7.22	14.44	7.00	0.50	1.11	1.03	123.24	1099.07

Fig. 3 Calculated minimum outer diameters of the barrel by Method 1 at  $Re = 1100$  MPa

In this method, according to the theory of the maximum elongation, it follows that the reduced stress on the inner surface of the wall is less (or underestimated compared to the real pressure) than in the shear stress method (at higher pressures). At lower pressures, the opposite is true.

**3.2 Method 2. According to the author J. Škvařek, the theory of the maximum elongation with a sample calculation at  $Re = 1100$  MPa [13]**

The tension of a simple gun barrel is solved under the assumption of a plane tension of the barrel wall (the tension in the prolonged direction is zero), due to the static pressure load of the gas. The relations for calculating the wall tension of the barrel derived from these assumptions give results that are suffi-

cient for both strength control and wall thickness design. If we assume that the barrel will be loaded only by the inner pressure  $p_1$ , then the reduced stress according to the theory of the maximum elongation is determined from the relation:

$$\sigma_{red}^I = \frac{2}{3} \cdot \frac{2 \cdot x^2 + 1}{a^2 - 1} \cdot p_1$$

- where  $a = r_2/r_1$  and  $x = r_2/r$ ,
- $r_1$  – inner radius of the barrel,
- $r_2$  – outer radius of the barrel,
- $r$  – radius of the barrel at the point of calculation,
- $\sigma_{red}$  = stress according to maximum elongation,
- $p = p_1$  – inner pressure.

When calculating the minimum wall thickness of the barrel, the yield strength will not be exceeded further than 0.5 mm from the inside of the barrel wall.

P.č.	$r_1$ (mm)	$r_2$ (mm)	$d_{min}$ (mm)	$r$ (mm)	$r-r_1$ (mm)	$a$	$x$	$p$ (MPa)	$s_{red}$ (MPa)
1	10.40	18.60	37.20	10.90	0.50	1.79	1.71	529.1	1094.77
2	7.165	12.50	25.00	7.665	0.50	1.74	1.63	529.1	1090.68
3	6.50	12.00	24.00	7.00	0.50	1.85	1.71	577.0	1098.53
4	6.50	7.22	14.44	7.00	0.50	1.11	1.03	123.24	1099.07

Fig. 4 Calculated minimum outer diameters of the barrel by Method 2 at  $Re = 1100$  MPa

In this method, according to the theory of the maximum elongation, it follows that the reduced stress on the inner surface of the wall is less (or underestimated compared to the real pressure) than in the shear stress method (at higher pressures). At lower pressures, the opposite is true.

**3.3 Method 3. According to the authors J. Pech and F. Kozderek with sample calculation at  $Re = 1100$  MPa [10]**

$$R = r \cdot \sqrt{\frac{k + 0,4 \cdot p}{k - 1,3 \cdot p}}$$

Formula for calculating the minimum wall thickness:

- $R$  – outer radius,
- $r$  – inner radius,
- $p_k = p$  – inner overpressure,
- $k$  – allowable stress (yield strength).

N1. Polásek, N2. Danko: Dependence of approximation with the maximum design pressure to the yield strength of the material and the applicability of various types of calculations to the minimum safety wall thickness of the gun barrel, calculations for caliber 50BMG

P.č.	r (mm)	d <sub>2MN</sub> (mm)	p (MPa)
1	10,40	36,75	529,1
2	7,165	27,89	529,1
3	6,50	25,17	577,0
4	6,50	14,36	123,24

Fig. 5 Calculated minimum outer diameters of the barrel by Method 3 at Re = 1100 MPa

This method is very practical. With this method it is possible to calculate favourable results, even if the results of other methods are already really unusable values (meaning very big minimum outer averages, which can no longer be used in practice). This method shows that it is older, from the war period, when the designers could quickly verify in practice what they calculated, and thus created more effective empirical methods than just theoretical way.

re=1100MPa, min. diameter	diameter 20,8 mm Pk 529,1 MPa	diameter 14,33 mm Pk 529,1 MPa	diameter 13 mm Pk 577 MPa	diameter 13 mm Pk 123,24 MPa
method				
1 [mm]	47,65	33,18	38,18	14,60
2 [mm]	40,00	27,60	27,60	14,80
3 [mm]	36,75	27,89	25,17	14,36
max. [mm]	47,65	33,18	38,18	14,80
min. [mm]	36,75	27,60	25,17	14,36
diameter difference max-min [mm]	10,90	5,58	13,01	0,44
radius difference max-min [mm]	5,45	2,79	6,51	0,22
difference max-min [%]	29,66	20,22	51,69	3,06

Fig. 6 Results of outer diameters of walls at Re = 1100 MPa, calculated by all methods

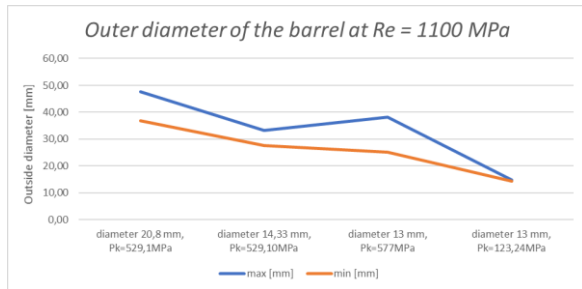


Fig. 7 Outer diameter of the barrel at Re = 1100 MPa

re=900MPa, min. priemery	diameter 20,8 mm Pk 529,1 MPa	diameter 14,33 mm Pk 529,1 MPa	diameter 13 mm Pk 577 MPa	diameter 13 mm Pk 123,24 MPa
method				
1 [mm]	99,99	69,83	N	14,95
2 [mm]	53,00	36,40	40,80	15,02
3 [mm]	47,15	36,31	35,46	14,74
max. [mm]	99,99	69,83	40,80	15,09
min. [mm]	47,15	36,31	35,46	14,65
diameter difference max-min [mm]	52,84	33,52	5,34	0,44
radius difference max-min [mm]	26,42	16,76	2,67	0,22
difference max-min [%]	112,07	92,32	15,06	3,00

Fig. 8 Results of outer diameters of walls at Re = 900 MPa, calculated by all methods

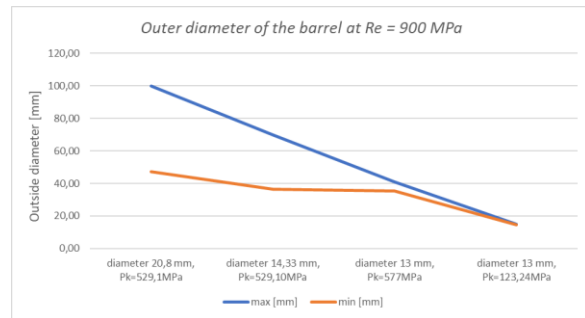


Fig. 9 Outer diameter of the barrel at Re = 900 MPa

re=800MPa, min. diameter	diameter 20,8 mm Pk 529,1 MPa	diameter 14,33 mm Pk 529,1 MPa	diameter 13 mm Pk 577 MPa	diameter 13 mm Pk 123,24 MPa
method				
1 [mm]	N	N	N	15,18
2 [mm]	72,80	50,20	82,00	15,32
3 [mm]	61,84	43,03	58,79	15,01
max. [mm]	72,80	65,17	82,00	15,32
min. [mm]	61,84	50,20	58,79	15,01
diameter difference max-min [mm]	10,96	14,97	23,21	0,31
radius difference max-min [mm]	5,48	7,49	11,61	0,16
difference max-min [%]	17,72	29,82	39,48	2,07

Fig. 10 Results of outer diameters of walls at Re = 800 MPa, calculated by all methods

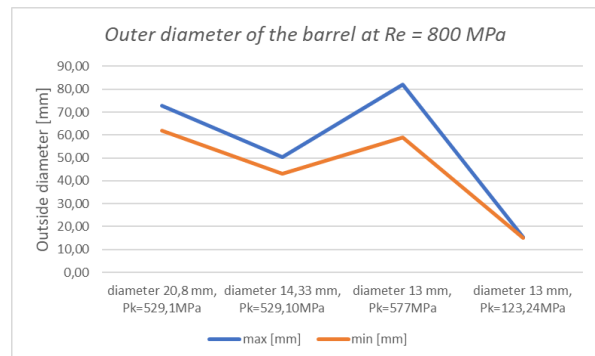


Fig. 11 Outer diameter of the barrel at Re = 800 MPa

## 4 Conclusion

When calculating the minimum wall thicknesses of the gun barrel, we must mainly take safety into account. Although other parameters are also essential when calculating the minimum wall thicknesses of the barrel for the standard cartridge where the maximum usable pressures are lower. For example, up to 200-300MPa, even with the inclusion of the reserve and when using standard barrel materials such as ČSN 15142 or ČSN 15230, there is no problem with calculating the minimum wall thickness of the barrel. This is because the yield strength of the material is much bigger than the maximum usable pressures at which the barrel is loaded during firing. If we design a gun barrel where the maximum usable pressures are higher than 300Mpa, with the standard usage of cartridge, there is a problem with calcula-

tions using standard materials such as ČSN 15142 or ČSN 15230. The design of a safe minimum wall thickness of the barrel requires materials with a higher yield strength while maintaining the minimum impact toughness. There are also problems in calculating the minimum wall thicknesses of the barrel, especially if the yield strength of the material is approaching the maximum usable pressures in the barrel. This is also seen in our work.

With a yield strength of 1100MPa, there was no problem to calculate the minimum diameters in each part of the barrel by all three methods. At a yield strength of 900MPa, there was no problem to calculate the minimum diameters in each part of the barrel by Methods 2 and 3, but Method 1 had a problem in calculating the minimum wall thickness of the barrel at a pressure of 577MPa and an inside diameter of 13mm. With a yield strength of 800MPa, there was no problem to calculate the minimum diameter in each part of the barrel by Methods 2 and 3, but Method 1 no longer gave us any result. Based on this work, calculation Methods 2 and 3 are more suitable when calculating the minimum required walls of the gun barrel. The best method is the method marked as number 3. At the smallest wall thicknesses, it also provides us with good safety. From our own experience we can write that even though this method calculates the smallest wall thicknesses, it provides sufficient security for the user of the weapon.

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