SIMULATION OF A MATHEMATICAL MODEL OF A WIND TURBINE

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Abstract
This paper presents a mathematical model of a wind turbine and its simulation. This is one of the main resources available to the island system (Grid-Off system).

Keywords: wind turbine, island system, Grid-Off system, renewable energy source.

1 Introduction to wind turbine

A wind turbine is basically a converter, or in other words a device that transforms one type of energy into another. In this case, it is the transformation of mechanical energy into electrical energy.

The source of mechanical energy is the flow (flow) of air, which acts on the turbine blades. The blades are located on a shaft which is coupled to a permanent magnet (magnet). The magnets are a rotating part, which is named the rotor. The stator consists of a coil (coils) of wound copper conductor. Due to the changing magnetic field (PM - permanent magnets), an electrical voltage is induced at the terminals (terminals) of the coil / coils. In essence, it is a synchronous generator, since the variable electric field is coupled (synchronized) with the speed of the changing and magnetic fields. [1-5]

A schematic block diagram of a wind turbine as a synchronous generator is shown in Fig. 1.

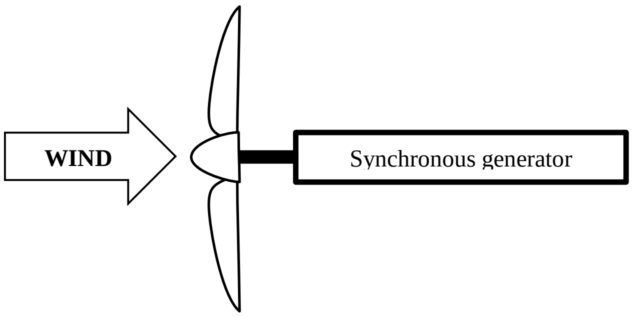

Fig. 1 *Principle block diagram of a wind turbine*

The kinetic energy of wind (air) is converted into mechanical energy of a rotating shaft. However, the validity of the continuity implies that the flow rate slows down on the turbine, since the air flow in front of the blades is intact with the velocity v and behind the blades the flow velocity is different. This fact is expressed by the power coefficient of the turbine C_p, which expresses the ratio of the mechanical power of the turbine P_m to the power of the intact air flow P_0, which is expressed by equation (1)

$$C_p = \frac{P_m}{P_0} \tag{1}$$

The maximum value of the turbine power coefficient is 0.59259, while physically feasible turbines have smaller values. [5-10]

We calculate the mechanical moment by the ratio of the mechanical power of the turbine to the angular velocity of the rotor ω_r, which we express mathematically as equation (2)

$$T_m = \frac{P_m}{\omega_r} = \frac{C_p \frac{1}{2}\rho A_r v^3}{\omega_r} = \frac{1}{2}\rho v^3 \frac{C_p A_r}{\omega_r} \tag{2}$$

where ρ is the air density and A_r is the rotor area. [11-12]

The mechanical connection of the wind turbine is such that if we proceed chronologically, the blades (blades) of the wind turbine are connected to the shaft to which the generator rotor is further connected. The blades generate a mechanical moment T_m. The shaft itself has a certain distributed weight, which is expressed by the moment of inertia, which depends on the time change of the angular velocity of the rotor. In terms of direction, the torque of the generator T_g (braking torque) acts in the opposite way, the mathematical expression of the balance of moments is represented by equation (3)

$$T_m = J_a \frac{d\omega_r}{dt} + T_g \tag{3}$$

From the point of view of creating a mathematical model, it is necessary to modify equation (3) into the form (4)

$$\frac{d\omega_r}{dt} = \frac{1}{J_a}\left(T_m - T_g\right) \tag{4}$$

Next we need to calculate the torque of the generator, which is expressed as (5)

$$T_g = \frac{3 I_g^2 L_s n_p}{2\tan(\delta)} \tag{5}$$

where I_g is the current flowing through the generator load, L_s is the stator inductance, n_p is the number of generator poles and the angle δ is between the internal generated voltage E_g and the voltage at the generator terminals U_g.

The internal voltage of the E_g generator depends on the force of the permanent magnet k_{PM}, the angular velocity of the rotor ω_r and the number of poles n_p, which is mathematically expressed as equation (6)

$$E_g = k_{PM}\omega_r \frac{n_p}{2} \tag{6}$$

If we connect the load R_z to the terminals of the generator, which is schematically shown in Fig. 2, we can express the equation for the generated voltage U_g at the terminals as equation (7) and the current I_g passing through the load as equation (8)

$$U_g = E_g \cos(\delta)\cos\left(\omega_g t\right) \tag{7}$$

$$I_g = \frac{U_g}{R_z} = \frac{E_g \cos(\delta)}{R_z}\cos\left(\omega_g t\right) \tag{8}$$

where ω_g is the angular frequency of the generator current.

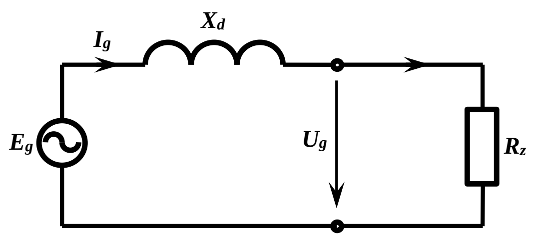

Fig. 2 *Connecting the load to the generator*

2 Mathematical model of wind turbine and its simulation

Based on the mathematical expressions in the previous section, a mathematical model will be constructed, which essentially consists of four basic blocks as shown in Fig. 3.

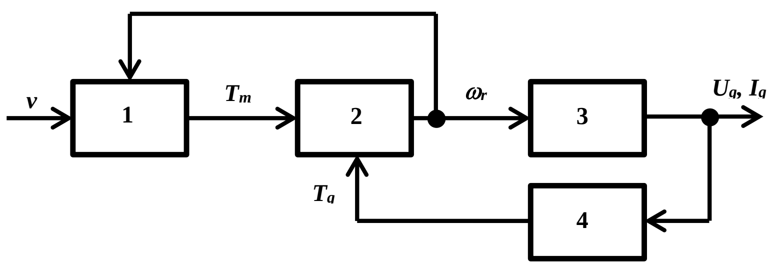

Fig. 3 *Block diagram of a mathematical model of a wind turbine*

The compilation of the model and its simulation was carried out in the SCILAB software, specifically in the XCos part, where individual blocks 1 to 4 as whole units (sub-blocks) were created and they contained individual blocks / parts of the scheme that expressed mathematical implementation as required, which is described in the following text.

Block 1 represents the incorporation of equation (2) into the model, where the inputs are the wind speed and the value of the angular velocity of the rotor. There are also constants in the model, which are summarized in Tab. 1. The model of block 1 is shown in Fig. 4.

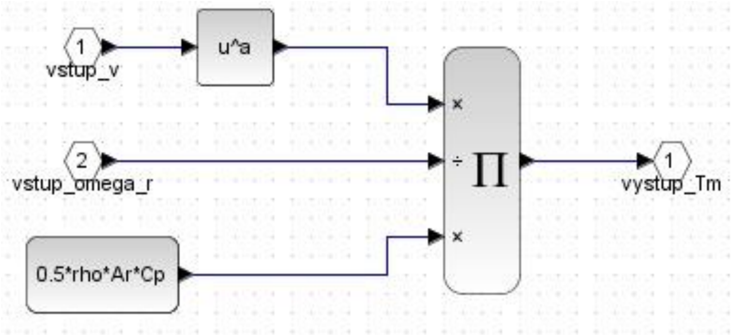

Fig. 4 *Block diagram of block 1 - the output is the mechanical torque T_m*

Block 2 after the adjustment expresses equation (4), the adjustment is necessary because the equation expresses the time change, where the inputs are the mechanical torque (output from block 1) and the generator torque, which is the output of block 4. The block diagram of block 2 is in Fig. 5.

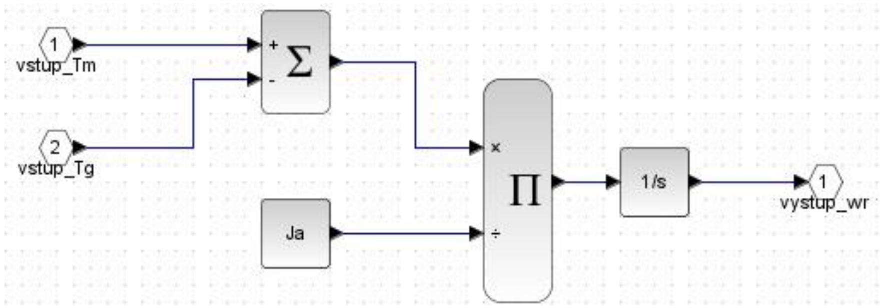

Fig. 5 *Block diagram of block 2 - the output is the angular speed of the rotor ω_r*

Block 3 includes equations (6), (7) and (8) and its output is the electrical quantities U_g and I_g. The block diagram that implements this is shown in Fig. 6.

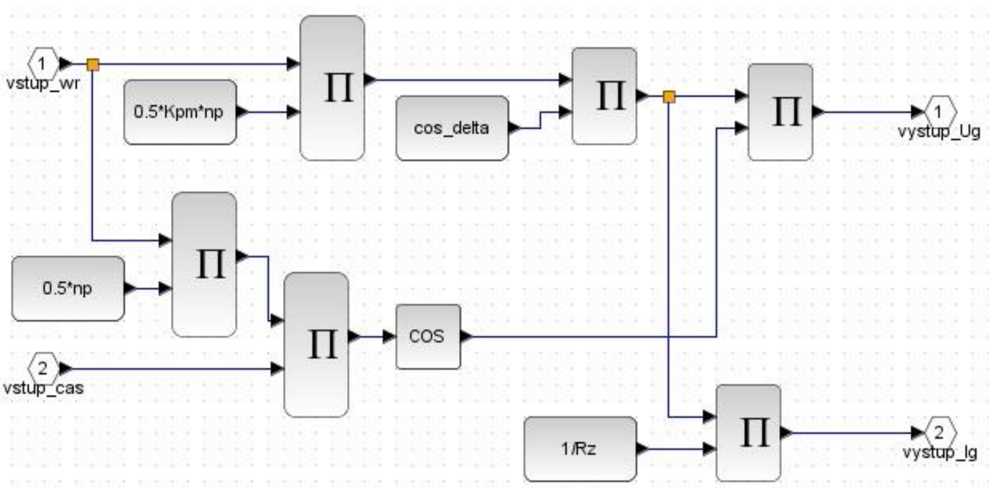

Fig. 6 *Block diagram of block 3 - the output are electrical quantities U_g and I_g*

Block 4 uses as input only the current flowing through the load I_g and the output is the torque of the generator, it is basically the realization of equation (5). The block diagram implementing Equation (5) is shown in Fig. 7.

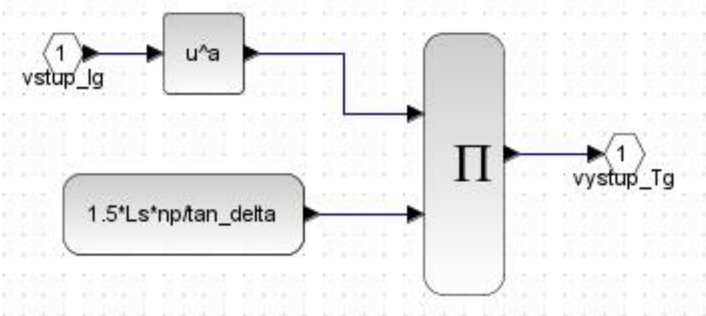

Fig. 7 *Block diagram of block 4 - the output is the torque of the generator T_g*

There were also constants in the individual parts of the model that need to be initialized at the beginning of the simulation. The specific values of the constants used are given in the following Table 1.

Table 1 *Stated values of constants used in the wind turbine model*

The name of the constant	*Value*	*Comment*
Kpm	10.3668 V.s	force PM
np	10	number of generator poles
delta	0.1745329 rad	angle between E_g and U_g
cos_delta	0.9848078	cosine angle delta
tan_delta	0.1763270	sine angle delta
Ls	3.07e-3 H	stator inductance
Ja	0.748 kg.m^2	the resulting moment of inertia
rho	1.29 kg.m^{-3}	air density
Rt	1.35 m	rotor radius
Ar	5.7255526 m^2	rotor surface
Cp	0.5	power factor
Rz	100 Ω	load resistance

The resulting block diagram represents the circuit according to Fig. 3, where the individual blocks are interconnected and also blocks for displaying waveforms are added. Such a complete connection of the wind turbine model is shown in Fig. 8.

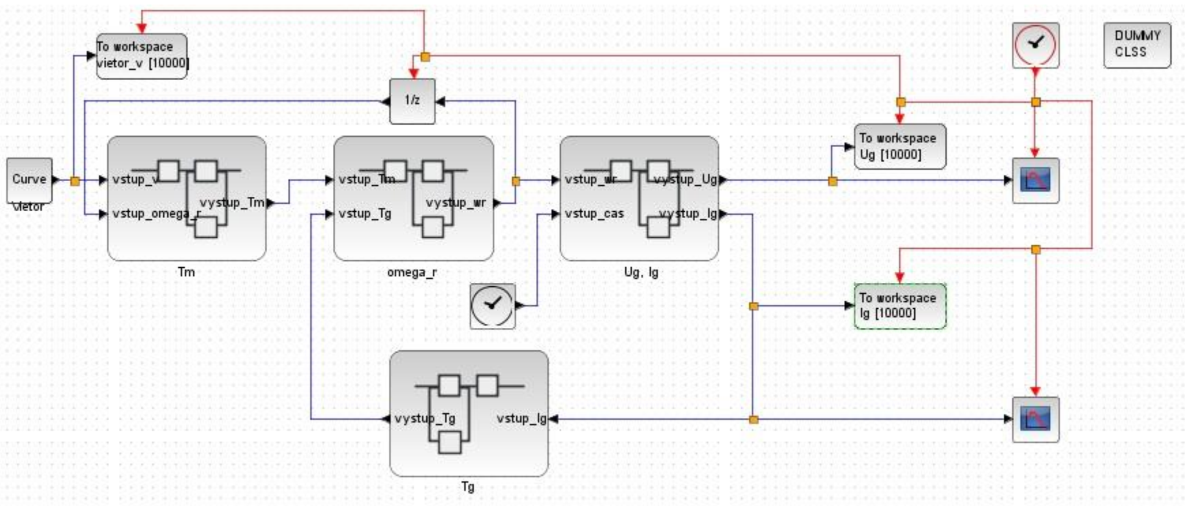

Fig. 8 *Final simulation model of a wind turbine*

In the given model shown in Fig. 8, a simulation was performed where the input to the model was a variable wind speed to which the wind turbine is exposed. The end load was a resistance with a value of 100 Ω and only the course of the generated current I_g passing through the load R_z was evaluated.

In Fig. 9 shows the simulated course of the generated current at the same time as the course expressing the change in wind speed (shown in blue), where it is possible to see the reaction to the change in current (shown in green) size from the change in wind speed. The value of wind speed is in m.s^{-1} and the value of current magnitude in A, since these values are close in size, therefore one scale on the y-axis is used.

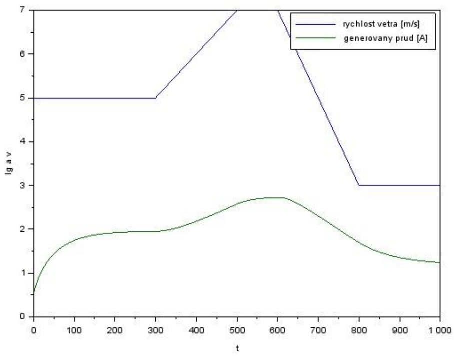

Fig. 9 *The simulated course of the generated Ig current*

3 Conclusion

In this paper, a mathematical model designed to simulate a wind turbine was designed and built. The current generated through the load was mainly simulated and presented. The input to the simulation was the variable magnitude of the wind speed acting on the wind turbine. The simulation shows that if we take I_{gmax} = 2.7 A and a load of 100 Ω, the maximum power was 730 W.

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