MATHEMATICAL MODEL OF WIND TURBINE IN GRID-OFF SYSTEM

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Abstract
This paper deals with the construction of a mathematical model of a wind turbine, which is one of the sources in the Grid-Off system.

Keywords: mathematical model, wind turbine, Grid-Off system, electric generator, wind conditions.

1 Introduction

As one of the power sources of the Grid-Off system is a wind turbine. It is advantageous to work with a mathematical model for the need of experimental research. In Fig. 1 is a schematic connection of a wind turbine to a container, which is a Grid-Off system. [1-4]

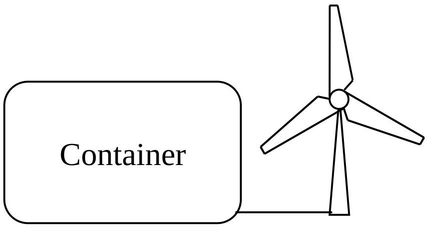

***Fig. 1** Wind turbine connection - Grid-Off system*

The process of producing electricity is, which in this case applies is as follows. The wind is carrying kinetic energy. Basically, the flow of air mass. The wind causes the propeller blades to rotate, so the kinetic energy is converted to mechanical energy. Subsequently, this mechanical energy is transformed into electrical energy by means of a suitable generator.

The mass flow energy passing through the area *S* determines the power of the intact air flow

$$P = \frac{1}{2}\rho S v^3 \tag{1}$$

where ρ is air density ($[\rho]$ = kg/m^3), *S* is area ($[S]$ = m^2) and *v* is velocity of flowing air ($[v]$ = m/s).

The mechanical energy that can be obtained by converting the kinetic energy of the air flow corresponds to the difference in power of the air flow upstream and downstream of the turbine, as shown in Fig. 2. The mathematical expression is as follows:

$$P = \frac{1}{2}\rho S_1 v_1^3 - \frac{1}{2}\rho S_2 v_2^3 \tag{2}$$

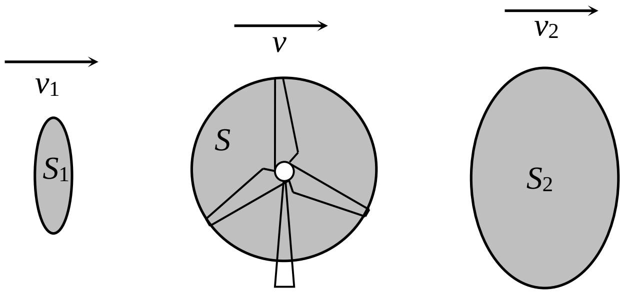

***Fig. 2** Air flow through turbine*

Based on the continuity equation is valid

$$\rho S_1 v_1 = \rho S_2 v_2 = \rho S_3 v_3 \tag{3}$$

We can then equation (2) using equation (3) shaped in the form

$$P = \frac{1}{2}\rho S_1 v_1 (v_1^2 - v_2^2) \tag{4}$$

The mechanical power of the wind turbine rotor can be expressed by relation (4) considering the momentum conservation law and its mathematical expression is

$$P = \frac{1}{4}\rho S v_1^3 \left(1 - \frac{v_2^2}{v_1^2} + \frac{v_2}{v_1} - \frac{v_2^3}{v_1^3}\right) = \underbrace{\frac{1}{2}\left(1 - \frac{v_2^2}{v_1^2} + \frac{v_2}{v_1} - \frac{v_2^3}{v_1^3}\right)}_{C_p} \underbrace{\frac{1}{2}\rho S v_1^3}_{P_w} \tag{5}$$

where C_p is dimensionless ratio of the extractable power P to the kinetic power P_w available in the undistributed stream. [4-5]

Maximum mechanical performance can only be achieved under certain conditions when the ratio v_2/v_1 is 1/3. In this case, the theoretical value of 59. 26 % is reached. [6]

The ratio of the peripheral speed of the rotor blade tip to the speed of the intact air flow upstream of the turbine is called turbine speed [7]

$$\lambda = \frac{\omega_r R_t}{v_1} \tag{6}$$

Next, we express the relation for the calculation of the mechanical moment T_m of the turbine rotor

$$T_m = \frac{P}{\omega_r} = \frac{C_p \frac{1}{2}\rho S v_1^3}{\omega_r} = \frac{1}{2}\rho S \frac{C_p v_1^3}{\frac{\lambda v_1}{R_t}} = \frac{1}{2}\rho S v_1^2 R_t \underbrace{\frac{C_p}{\lambda}}_{C_q} = \frac{1}{2}\rho S v_1^2 R_t C_q \tag{7}$$

where C_q is turbine rotor torque coefficient. [8-9]

2 Synchronous generator

The transformation of mechanical energy into electrical energy is carried out by means of a wind turbine, which is a synchronous generator with permanent magnets. The schematic diagram of the generator is shown in Fig. 3.

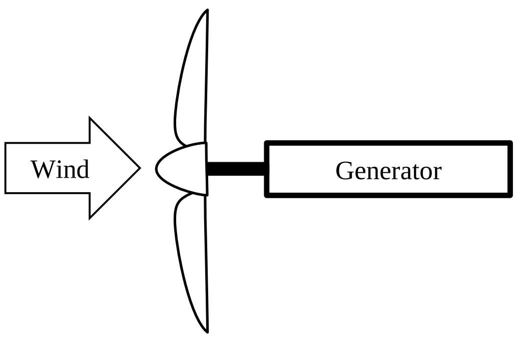

Fig. 3 *Transformation of mechanical energy into electrical energy*

The wind turbine includes blades that rotate the shaft on which the rotor of the permanent magnet synchronous generator is mounted. The rotor is formed by a permanent magnet, which causes a time-varying magnetic field, which results in voltage generation in the stator winding.

A schematic representation of a synchronous generator with a rotor of a permeable magnet and a stator formed by a winding is shown in Fig. 4.

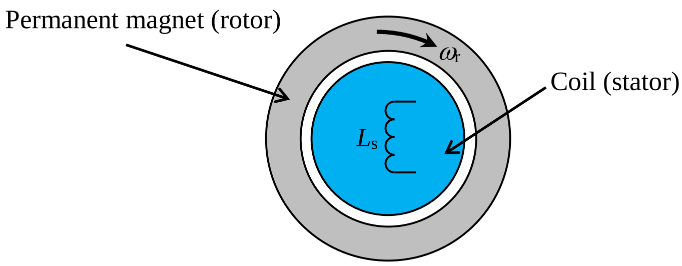

Fig. 4 *A schematic representation of a synchronous generator*

Depending on the torque balance condition, the dynamic system applies

$$T_m = T_g + J_a \frac{d\omega_r}{dt} \tag{8}$$

where T_m is the turbine torque produced by the wind on its blades, T_g is the braking torque of the generator with the load acting against the mechanical torque of the turbine and J_t is the moment of inertia of the aggregate. If we use the equation (7) and equation (8) we can adapt to the shape

$$\frac{d\omega_r}{dt} = \frac{T_m - T_g}{J_a} = \frac{\frac{1}{2}\rho S v_1^2 R_t C_q - T_g}{J_a} \tag{9}$$

The permanent magnets in the rotor, by their rotation around the stator, induce the internal voltage of the generator, for which

$$E_g = \mathrm{K}_{pm} \frac{\omega_r n_p}{2} \tag{10}$$

where K_{pm} is constant, which expresses the magnetic field strength of the permanent magnets of the generator, n_p is the number of poles of the generator and ω_r is the angular velocity of the rotor. [10-13]

If X is the reactance of the generator coil ($X = \omega_g L_s$, $\omega_g = n_p\omega_r/2$), and δ is angle between $\boldsymbol{U}_g$ and $\boldsymbol{E}_g$, the equation applies to the terminal voltage $\boldsymbol{U}_g$

$$\boldsymbol{U}_g = \boldsymbol{E}_g - \mathrm{j}X\boldsymbol{I}_g = |\boldsymbol{E}_g|\mathrm{e}^{j\delta} - X|\boldsymbol{I}_g|\mathrm{e}^{j\frac{\pi}{2}} = E_g\cos(\delta) + \underbrace{\mathrm{j}E_g\sin(\delta) - \mathrm{j}XI_g}_{0} = E_g\cos(\delta) \tag{11}$$

where

$$E_g \sin(\delta) = XI_g \tag{12}$$

The active power of the generator can be expressed as

$$P_g = \frac{3E_g U_g}{X}\sin(\delta) \tag{13}$$

Equation (13) using equation (11) and (10) can be modified to shape

$$P_g = \frac{3E_g E_g}{X}\underbrace{\cos(\delta)\sin(\delta)}_{\frac{1}{2}\sin(2\delta)} = \frac{3E_g^2}{2X}\sin(2\delta) = \frac{3\left(\mathrm{K}_{pm}\frac{\omega_r n_p}{2}\right)^2}{2X}\sin(2\delta) = \frac{3}{8X}\left(\mathrm{K}_{pm}\omega_r n_p\right)^2 \sin(2\delta) \tag{14}$$

If we consider equation (12), then equation (14) can be expressed as

$$P_g = \frac{3E_g^2}{X}\cos(\delta)\sin(\delta) = \frac{3\left(\frac{XI_g}{\sin(\delta)}\right)^2}{X}\cos(\delta)\sin(\delta) = \frac{3I_g^2 X}{\tan(\delta)} \tag{15}$$

Generator moment T_g can be expressed depending on which equation we use, whether (15) or (14)

$$T_g = \frac{P_g}{\omega_r} = \frac{3I_g^2 X}{\omega_r \tan(\delta)} = \frac{3I_g^2 L_s n_p}{2\tan(\delta)} \tag{16}$$

$$T_g = \frac{P_g}{\omega_r} = \frac{3}{8X\omega_r}\left(K_{pm}\omega_r n_p\right)^2 \sin(2\delta) \tag{17}$$

Based on these mathematical expressions, it is possible to construct a mathematical model by considering the particular parameters of a real wind turbine in the equations. The input parameter is the wind speed and the electrical output generated. It is also possible to add a connected load or use a more sophisticated wind model. Also in the model could be expressed regulation of wind turbine blades, possibly supplemented generator with speed control (angular speed).

3 Conclusion

In this paper is analyzed generator of electric energy, which is formed by wind turbine. It's about using that generator in the Grid-Off system. Mathematically, the continuity and continuity between the kinetic energy conversion that is contained in the wind and converted to the mechanical energy of the turbine are expressed. Subsequently, this mechanical energy is transformed into electrical energy by means of a synchronous permanent magnet generator.

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