WIND TURBINE IN GRID-OFF SYSTEM

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Abstract
The paper presents an analysis of a wind turbine, which is one of the sources of electricity in the Grid-Off system.

Keywords: wind turbine, Grid-Off system, electric generator, Photovolaic panel, wind conditions.

1 Introduction

The basic concept of a grid-off system consists of different sources that supply electricity. Mainly they are photovoltaic panels and wind turbine or even a diesel generator is still considered. This basic concept of grid-off system is shown in Fig. 1.

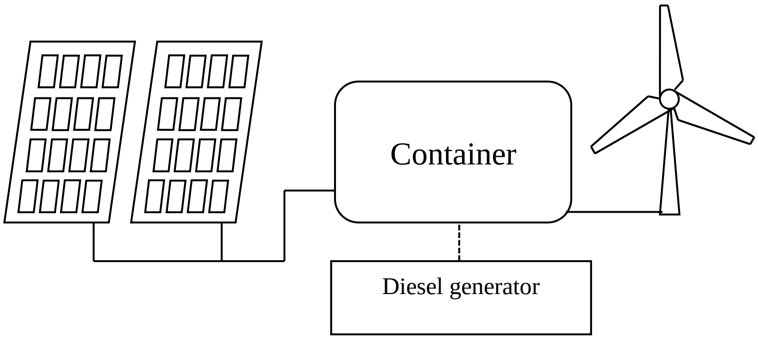

Fig. 1 *The basic concept of a Grid-Off system*

Another additional source is the thermoelegtric generator (TEG), which is based on the Seebeck effect. This TEG is part of photovoltaic panels. [1][2][3][4]

2 Wind power source

In this section, factors that affect wind turbine performance will be listed. The wind turbine performs a rotational movement based on the acting wind, which has kinetic energy E_k. The mathematical expression (1) is:

$$E_k = \frac{1}{2} m v^2 \tag{1}$$

where E_k is kinetic energy ($[E_k] = J$), m is mass ($[m] = kg$) and v is air speed $[v] = m/s$.

In Fig. 2, the kinetic energy contained in the air element is located.

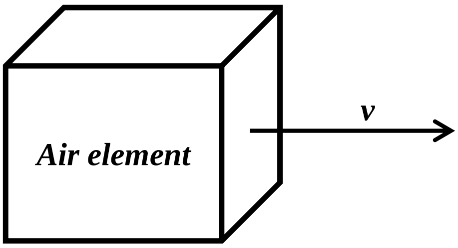

Fig. 2 *The kinetic energy contained in the air element with mass m*

Furthermore, we can express power P, which represents the energy per time (2). The mathematical expression is:

$$P = \frac{E_k}{t} = \frac{1}{2}\frac{m}{t} v^2 = \frac{1}{2} s v^2 \tag{2}$$

where s is mass flow ($[s]$ = kg/s).

Furthermore, it is possible to consider the wind time flow, which represents the flow of quantity over the area per unit of time, which is graphically shown in Fig. 3.

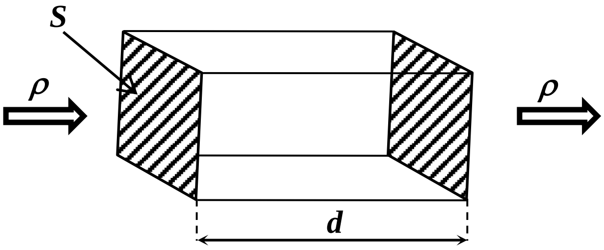

Fig. 3 *The wind time flow*

The mathematical expressed time flow of the wind will be

$$\frac{\rho}{t} = \frac{\frac{m}{V}}{t} = \frac{m}{Vt} = \frac{m}{Sdt} = \frac{1}{Sd}\frac{m}{t} = \frac{1}{Sd}s \tag{3}$$

where ρ is air density ($[\rho]$ = kg/m^3), wind time flow in volume $V = Sd$ ($[V]$ = m^3).

Next, from Equation (3) we express

$$s = \frac{\rho}{t}Sd = \rho S\frac{d}{t} = \rho Sv \tag{4}$$

If we substitute equation (4) into equation (2), we get

$$P = \frac{1}{2}sv^2 = \frac{1}{2}\rho Svv^2 = \frac{1}{2}\rho Sv^3 \tag{5}$$

In equation (5), the area S represents the area described by the wind turbine blades.

3 The maximum wind turbine power

Betz's law indicates the maximum power that can be extracted from the wind, independent of the design of a wind turbine in open flow. The law is derived from the principles of conservation of mass and momentum of the air stream flowing through an idealized "actuator disk" that extracts energy from the wind stream. [5]

In Fig. 4 is a schematic diagram of a constant density flow.

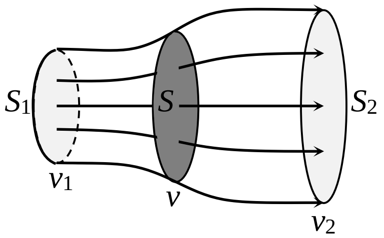

Fig. 4 *Schematic of fluid flow through a disk-shaped actuator*

Considering equation (4), for Fig. 4 can be expressed mathematically:

$$s = \rho S_1 v_1 = \rho S v = \rho S_2 v_2 \tag{6}$$

where v_1 is the speed in the front of the rotor, v_2 is the speed downstream of the rotor, v is the speed at the fluid power device, ρ is the fluid density, the area of the turbine is given by S, and S_1 and S_2 are the areas of the fluid before and after reaching the turbine. [5-8]

The force exerted on the wind by the rotor is the mass of air multiplied by its acceleration. In terms of the density, surface area and velocities, this can be written as

$$F = m\frac{dv}{dt} = \frac{\overbrace{m}^{s}}{t}\Delta v = s\Delta v = \rho S v(v_1 - v_2) \tag{7}$$

The power (rate of work done) of the wind is

$$P = \frac{dE}{dt} = \frac{Fdx}{dt} = F\frac{\overbrace{dx}^{v}}{dt} = Fv = \rho S v^2(v_1 - v_2) \tag{8}$$

However, power can be computed another way, by using the kinetic energy. Applying the conservation of energy equation to the control volume yields

$$P = \frac{\Delta E}{\Delta t} = \frac{1}{2}s(v_1^2 - v_2^2) \tag{9}$$

Looking back at the continuity equation, a substitution for the mass flow rate yield

$$P = \frac{1}{2}\rho S v(v_1^2 - v_2^2) \tag{10}$$

If we compare equations (8) and (10), we get

$$\begin{aligned}
&\rho S v^2(v_1 - v_2) = \frac{1}{2}\rho S v(v_1^2 - v_2^2) \\
&v(v_1 - v_2) = \frac{1}{2}(v_1^2 - v_2^2) \\
&v(v_1 - v_2) = \frac{1}{2}(v_1 - v_2)(v_1 + v_2) \\
&v = \frac{1}{2}(v_1 + v_2) \qquad (11)
\end{aligned}$$

If we substitute equation (11) to (10) and then modify it, we get

$$P = \frac{1}{2}\rho S\overbrace{\frac{1}{2}(v_1 + v_2)}^{v}(v_1^2 - v_2^2) = \frac{1}{4}\rho S v_1^3\left(1 - \frac{v_2^2}{v_1^2} + \frac{v_2}{v_1} - \frac{v_2^3}{v_1^3}\right) \tag{12}$$

If we express the wind power before the turbine as

$$P_w = \frac{1}{2}\rho S v_1^3 \tag{13}$$

Then, the equation (12) can be adjusted to shape

$$P = \overbrace{\frac{1}{2}\rho S v_1^3}^{P_w}\overbrace{\frac{1}{2}\left(1 - \frac{v_2^2}{v_1^2} + \frac{v_2}{v_1} - \frac{v_2^3}{v_1^3}\right)}^{C_p} = C_p P_w \tag{14}$$

where

$$C_p = \frac{1}{2}\left(1 - \frac{v_2^2}{v_1^2} + \frac{v_2}{v_1} - \frac{v_2^3}{v_1^3}\right) = \frac{P}{P_w} \tag{15}$$

is dimensionless ratio of the extractable power *P* to the kinetic power P_w available in the undistributed stream.

Next we will look for the extreme coefficient C_p as a function of $\frac{v_2}{v_1}$, coefficient $C_p(\frac{v_2}{v_1})$ [9-12]

$$\left.\frac{dC_p}{d\xi}\right|_{\xi = \frac{v_2}{v_1}} = \frac{d}{d\xi}\left(\frac{1}{2}(1 - \xi^2 + \xi - \xi^3)\right) = 0 \tag{16}$$

$$\begin{aligned}
&\frac{1}{2}(-2\xi + 1 - 3\xi^2) = 0 \\
&-\frac{3}{2}\left(\frac{2\xi}{3} - \frac{1}{3} + \xi^2\right) = 0 \\
&\left(\xi - \frac{1}{3}\right)(\xi + 1) = 0 \qquad (17)
\end{aligned}$$

The mathematical solution of equation (17) is the values of 1/3 and -1, while the real value of $\xi=1/3$ is real where the maximum value of C_p is reached is actually applicable:

$$C_p\Big|_{\frac{v_2}{v_1}=\frac{1}{3}} = \frac{1}{2}\left(1-\left(\frac{1}{3}\right)^2+\frac{1}{3}-\left(\frac{1}{3}\right)^3\right) = \frac{1}{2}\left(1-\frac{1}{9}+\frac{1}{3}-\frac{1}{27}\right) = \frac{1}{2}\frac{32}{27} = \frac{16}{27} \tag{18}$$

Then we can express equation (19) for the maximum value of C_p, which actually represents the maximum wind energy

$$P = \frac{16}{27}\frac{1}{2}\rho S v_1^3 = \frac{8}{27}\rho S v_1^3 \tag{19}$$

If we express it as a percentage, we are adding value

$$C_{p\,max} = \frac{16}{27} \cong 0.5926 = 59.26\ \% \tag{20}$$

4 Conclusion

The Grid-Off system is self-sufficient in terms of energy. In this paper, attention is paid to one power source, the wind turbine, as a power source for the Grid-Off system. Furthermore, the individual parameters are discussed, by means of which it is possible to maximize the output of the wind turbine, which is applicable either in the design of the turbine or only in the modification of the already used turbine. In theory, the maximum value we can get from wind energy is 59. 26 %.

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References

[1] Jus, M.: Využitie termoelektrického efektu v grid-off fotovoltických systémoch. In: Výzbroj a technika ozbrojených síl 2019 : 25. medzinárodná vedecká konferencia. - Liptovský Mikuláš : Akadémia ozbrojených síl generála Milana Rastislava Štefánika, 2019. - ISBN 978-80-8040-585-4. - s.83-88.

[2] Jus, M.: Thermoelectric generator model and simulation. In: Transfer 2019 : 20th International Scientific Conference. - Trenčín : TnUAD, 2019. - ISBN 978-80-8075-889-9. - s.71-78, CD ROM.

[3] Jus, M., Balla, J.: Parametric model of thermoelectric generator. In: University review. - ISSN 1339-5017. - Roč.13, č.3(2019), s.40-45.

[4] [02.03.2020], https://en.wikipedia.org/wiki/Thermoelectric_effect

[5] [02.03.2020], https://en.wikipedia.org/wiki/Betz%27s_law

[6] Ragheb, M., Ragheb, A. M.: Wind turbines theory—The Betz equation and optimal rotor tip speed ratio, fundamental and advanced topics in wind power. In R. Carriveau (ed.), Fundamental and Advanced Topics in Wind Power, InTech, 2011.

[7] [02.03.2020], https://www.fei.stuba.sk/buxus/docs/2019/AUTOREFERAT_Zuscak.pdf

[8] Zuščák, J., Janíček, F., Kijan, V., Váry, M.: Dynamic behavior of self-excited induction generator in Off-grid operation under resistive load. In Power engineering 2018. Control of Power Systems 2018 : 13th International scientific conference. Tatranské Matliare, Slovakia. June 5-7, 2018. 1. vyd. Bratislava : Slovak University of Technology, 2018, S. 89-94. ISBN 978-80-89983-00-1.

[9] Hansen, M.O.L.: Aerodynamics of Wind Turbines, 3rd ed., Routledge, 2015, ISBN 978-1138775077.

[10] Zuščák, J., Janíček, F.: Calculation of wind turbine power using Matlab. In ELITECH´16 [elektronický zdroj] : 18th Conference of doctoral students. Bratislava, Slovakia. June 8, 2016. 1. vyd. Bratislava : STU, 2016, CD-ROM, [5] s. ISBN 978-80-227-4561-1.

[11] [03.03.2020], https://en.wind-turbine-models.com/turbines/13-vestas-v47

[12] [03.03.2020], https://globalwindatlas.info/